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Theory of Clarifier Operation. I. Quiescent-Hindered Settling of Flocculating Slurries

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Theory of Clarifier Operation. I. Quiescent-Hindered Settling of Flocculating Slurries

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Abstract

The operation of quiescent (batch) clarifiers operating in the Class III mode (hindered settling) with flocculating slurries is modeled by means of a set of nonlinear partial differential equations, the continuity equations for the individual species of particles. Disintegration of the larger particles by viscous drag forces is assumed to be first-order. The equations are integrated numerically for the case of batch settling (such as is used in jar tests), and the dependence of settling characteristics on the parameters of the model is studied.

INTRODUCTION

Clarifiers and thickeners represent one of the most common types of equipment used in the processing of minerals and the treatment of industrial and municipal wastewaters. In the activated sludge process, settled wastewater is mixed with microorganism-rich sludge recycled from a clarifier for biological oxidation. At the present time, precipitation and flocculation followed by clarification is the most widely used class of methods for the removal of toxic metals from waste streams (*1–4*). Clarifiers operate in four modes (*5*): Class I, in which nonflocculating particles

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in quite dilute suspensions settle independently of each other, usually according to Stokes' law; Class II, in which the particles' settling motion is that of independent, freely falling particles, but in which collisions between particles may result in agglomeration, first analyzed by Smoluchowski (*6*, *7*); Class III, or hindered settling, zone settling, or blanket settling, in which (possibly agglomerating) particles are sufficiently close together that one can no longer describe their motion as independent free-fall in a viscous medium; and Class IV, or compression settling, in which water is slowly squeezed up through the narrow interstices of a relatively dense floc by virtue of the weight of the overlying floc. We shall be concerned with Class III operation, with Classes I and II included as limiting cases of the more general analysis.

Fair, Geyer, and Okun (*8*) give a good discussion of hindered settling, including its relation to filter backwashing, and the effect of viscous shear forces in causing the particles to break up on reaching a limiting size. Fitch, in McCabe and Eckenfelder's book (*9*), describes the four types of settling and gives a qualitative analysis of zone settling. Canale and Borchardt discuss zone settling in some detail, with engineering design formulas, but no mathematical model of the process (*5*). Eckenfelder (*10*), Eckenfelder and Thackston (*11*), and Metcalf and Eddy, Inc. (*12*) discuss the topic, concluding that heavy reliance on batch settling data is necessary, inasmuch as detailed analysis appears to be intractable. Early papers of interest in that they introduced new, more sophisticated concepts into the field of sedimentation include Hazen (*13*), Coe and Clevinger (*14*), Smoluchowski (*6*, *7*), Camp (*15*), and Steinour (*16*).

The modern approach to the theory of Class III sedimentation dates back to the mathematician Kynch (*17*); he assumed that the velocity of fall of a particle depends only on the local particle concentration, and then made use of the continuity equation. He was able to demonstrate the formation of layers where the particle density abruptly changes its value. Despite the fact that computational limitations forced Kynch to restrict himself to monodisperse systems without flocculation, it is clear from subsequent work that this paper was of major importance in that it inspired new approaches and efforts in the field. Dick and Ewing found that, contrary to Kynch's assumption, the initial settling velocity of biological sludges does depend (weakly) on tank depth, although sand settling velocities are independent of tank depth, in agreement with Kynch's postulate (*18*). Talmage and Fitch (*19*) and Fitch (*20*) also observed that Kynch's postulate is not valid as one approaches Class IV conditions (compression) and noted that the theory does not include flocculation, an important factor. Shannon and his co-workers in a series of papers examined batch and continuous thickening of spheres without flocculation; they also showed that the intersection of a rising concentration gradient with the clear fluid–slurry interface is responsible for nonlinearity in settling curves (*21–23*).

Scott, examining mathematical models for the mechanism of thickening, presented a useful formula for calculating the lab velocity of a falling particle with a boundary layer of bound water in terms of the solids concentration (*24*). The importance of this boundary layer of water was previously noted by Foust et al. (*25*). Vand published a formula for the viscosity of a suspension in terms of solvent viscosity, solids volume fraction, and a shape factor (*26*); this was an extension of earlier work by Einstein (*27*). Comings and co-workers developed a general expression for the hindered settling capacities from measured settling rates and slurry properties (*28*). They pointed out that the density and viscosity of the slurry, rather than those of the liquid, should be used. McLaughlin carried out a general analysis of the settling of suspensions in terms of the equations of continuity; he included flocculation and diffusion effects but did not pursue the differential equations in detail (*29*). Gaudin and Fuerstenau's work is concerned with compression (Class IV) settling, but their model of fluid flow through tubes and tubules in the slurry appears to lend itself well to analysis of fluid flow through a sludge blanket in Class III operation (*30*). Thomas has related hindered settling floc characteristics to rheological parameters and has published experimental data on the behavior of non-Newtonian suspensions (*31*, *32*). Tarrer and co-workers combined Kynch's theory for hindered batch settling with a plug flow fluid pattern to compute the effect of sludge blanket height and solids residence time on the thickening capacity of continuous flow clarifiers (*33*). Reimann and Menschel presented a derivation of the settling rate expression for nonflocculating suspensions of spherical, monodisperse particles for porosities (voids) of 0.3 to 0.6; they found good agreement with the experimental data (*34*).

Tracy and Keinath (*35*) published an analysis of a dynamic model for the thickening of activated sludge; the model incorporates all the operating parameters pertaining to the thickening function of a secondary clarifier but does not appear to take flocculation and particle disruption into account. The paper also contains a good review of the literature up to 1972. Gemmell (*36*) discussed the principles and theory as they bear on the mathematical modeling and simulation of the aggregation of suspended particles. Dick and Suidan (*37*) discussed the simulation of clarification and thickening, and presented a well-documented computer program for use in such modeling.

Davies et al. (*38*) and Carstensen et al. (*39*) have both noted the marked effect of zeta potential on hindered settling rates. Davies suggests that maximum reduction of surface polarity without flocculation is the most efficient condition for gravity settling. Studies on the hindered settling of barium sulfate (*40*); calcium carbonate in *p*-xylene (*41*); calcium carbonate in water (*42*); quartz, magnetite, and coal (*43*); titanium dioxide (*44*); and polystyrene spheres in cyclohexanol at various Reynolds numbers (*45*) give a sampling of the experimental data available beyond what has been cited above.

Interestingly enough, the detailed experimental studies of Barnea and Mizrahi (*46*) on settling liquid–liquid dispersions show no evidence of rising droplet concentration gradients. Dixon (*47*) has recently claimed that the occurrence of such gradients can only be interpreted as indicating physical contact between the particles—i.e., Class IV settling. We suspect that Dixon's neglect of inertial forces and the finite thickness of the liquid boundary layers around the particles may make the analysis not quite as straightforward as he indicates. His work certainly poses a challenge to the conclusions of Coe and Clevinger (*14*), Kynch (*17*), and many other workers in the field, and is in major part responsible for our interest in modeling Class III settling.

ANALYSIS OF BATCH SETTLING

We analyze the operation of quiescent batch clarifiers; this is relevant to the commonly used batch settling tests used in preliminary studies, and with minor modification it can be made to describe the operation of rectangular clarifiers, to be discussed in a later paper.

The continuity equation for coalescing and disintegrating particles we take to be

$$\begin{aligned}\frac{\partial c_n}{\partial t}(x, t) = &\sum_{j=1}^{[n/2]} c_j c_{n-j} |v_j - v_{n-j}| \pi (r_j + r_{m-j})^2 \\ &- \sum_{j=1}^{N-n} c_n c_j |v_n - v_j| \pi (r_n + r_j)^2 - \frac{\partial}{\partial x}(v_n' c_n) \\ &+ \sum_{m=n+1}^{N} k_{n,m-n}^{m} c_m (1 + \delta_{n,m-n}) - \sum_{j=1}^{[n/2]} k_{j,n-j}^{n} c_n, \\ &\qquad n = 1, 2, \ldots \quad (1)\end{aligned}$$

Here c_k = concentration of k-particles, aggregates of k elementary particles

$v_k = \frac{2[\Delta\rho(\mathbf{c})]grk^2}{9\eta(\mathbf{c})}$ = velocity of a k-particle relative to the surrounding liquid, calculated from Stokes' law (assumes low Reynolds numbers)

r_k = radius of a k-particle, assumed spherical, $= \left(\frac{3kV_1}{4\pi}\right)^{1/3}$

V_1 = volume of an elementary particle

V_k = volume of a k-particle, $= kV_1$

$\eta(\mathbf{c})$ = viscosity, a function of particle concentrations

$\Delta\rho(\mathbf{c})$ = difference in density between a particle and the surrounding slurry, a function of particle concentrations

v_k' = velocity of a k-particle relative to the laboratory

v'' = velocity of liquid relative to the laboratory

t = time

x = distance down from the top of the clarifier

$k^m_{n,m-n}$ = rate constant for the disruption of an m-particle into an n-particle and an $(m - n)$-particle

$\mathbf{c} = (c_1, c_2, c_3, \ldots)$ = concentration vector

g = gravitational constant

$\delta_{n,j} = 1$ if $n = j$, $= 0$ if $n \neq j$

$[n/2]$ = greatest integer $\leq n/2$

N = maximum number of elementary particles which may agglomerate to form a composite particle

In Eq. (1), the first two summations on the right-hand side describe the coalescence of particles through collisions; the next term, the divergence of the particle flux due to free fall through the liquid; and the last two summations, the disruption of composite particles through viscous drag forces.

In quiescent operation the volume flow of solids down is equal to the volume flow of liquid up, which yields

$$\sum_n v_n' c_n V_n + v''\left(1 - \sum_n c_n V_n\right) = 0 \tag{2}$$

so

$$v'' = \frac{-\sum_n v_n' c_n V_n}{1 - \sum_n c_n V_n} \tag{3}$$

Now $v_k' = v_k + v''$, and the v_k are known functions of $\mathbf{c}$; we need the v_k' and v'' in terms of the v_k.

$$v'' = \frac{-\sum_n (v_n + v'')c_n V_n}{1 - \sum_n c_n V_n} \tag{4}$$

from Eq. (3), from which

$$v'' = -\sum_n v_n c_n V_n \tag{5}$$

and

$$v_k' = v_k - \sum_n v_n c_n V_n \tag{6}$$

It is easily shown that

$$\Delta\rho(\mathbf{c}) = (\rho_s - \rho_l)(1 - \sum_n c_n V_n) \tag{7}$$

where ρ_s = density of solid particles
ρ_l = density of liquid

We use Einstein's formula *(27)* or one of the more elaborate formulas due to Vand *(26)* to calculate the viscosity of the slurry:

$$\eta = \eta_0(1 + 2.5c) \qquad \text{(Einstein)} \tag{8}$$

$$\log \frac{\eta}{\eta_0} = \frac{2.5c + 2.7c^2}{1 - 0.609c} \tag{9}$$

$$\eta/\eta_0 = 1 + 2.5c + 7.349c^2 + \cdots \tag{10}$$

$c = \sum_n c_n V_n$, the volume solids concentration

These formulas all assume spherical particles.

We estimate the disruption rate constants by assuming that they increase proportionally to the viscous drag force on the particle, given for an m-particle by $mV_1g(\Delta\rho)$, and that they are proportional to the number of ways in which the m elementary particles can be partitioned into two groups, one of n-particles and the other of $m - n$-particles, $m![n!(m - n)!]$. Thus, with some trepidation, we take

$$k^m_{n,m-n} = \kappa(\Delta\rho)\frac{mm!}{n!(m - n)!} \tag{11}$$

where κ is a proportionality constant to be assigned a value large enough to prevent composite particles from agglomerating to unrealistically large sizes.

This completes the specification of parameters and functions in Eq. (1). We next assign values to $\mathbf{c}(x, 0)$, the initial concentration distribution, and then integrate Eq. (1) forward in time, noting that our boundary conditions are zero flux at the top and bottom of the clarifier. This will be done by means of a predictor-corrector method which we have used previously (*48, 49*) and found to be quite stable. We choose a discrete set of equally spaced values of x; $x_m = (m - \frac{1}{2})\Delta x$, $m = 1, 2, \ldots, M$. This mesh approach converts Eq. (1) into a set of nonlinear first-order ordinary differential equations, as follows. We let $c_n(x_m, t) = c(n, m, t)$. Then

$$\begin{aligned}\frac{\partial c}{\partial t}(n, m, t) = {} & \frac{1}{\Delta x}[-v'(n, m, t)c(n, m, t) + v'(n, m-1, t)c(n, m-1, t)] \\ & + \sum_{d=1}^{[n/2]} c(n, m, t)c(n-j, m, t)|v(n, m, t) - v(n-j, m, t)| \\ & \times \pi(r_j + r_{n-j})^2 \\ & - \sum_{j=1}^{N-n} c(j, m, t)c(n, m, t)|v(n, m, t) - v(j, m, t)|\pi(r_j + r_n)^2 \\ & + \sum_{j=n+1}^{N} k^j_{n,j-n}c(j, m, t)(1 + \delta_{n,j-n}) \\ & - \sum_{j=1}^{[n/2]} k^n_{j,n-j}c(n, m, t) \qquad n = 1, 2, \ldots, N \\ & \qquad\qquad\qquad\qquad\qquad m = 1, 2, \ldots, M\end{aligned} \tag{12}$$

M = number of slabs into which the clarifier is partitioned.

The boundary condition at the top of the clarifier corresponds to the elimination of particle flux into the top slab (of thickness Δx) from above (the second term in the square bracket of Eq. 12). The boundary condition at the bottom of the clarifier corresponds to elimination of particle flux from the bottom of the bottom slab—deletion of the term $-v'(n, 1, t)c(n, 1, t)/\Delta x$ from Eq. (12), $m = 1$.

We now have specified all of the $dc(n, m, t)/dt$ as functions of $\{c(i, j, t)\} \equiv \hat{c}(t)$, the array of all the particle concentrations at the various values of x. We write this more compactly as

$$\frac{dc}{dt}(n, m, t) = f[\hat{c}(t)] \tag{12'}$$

The integration of Eq. (12′) is then done by the following predictor-corrector algorithm. Predictor (starter formula):

$$c^*(n, m, \Delta t) = c(n, m, 0) + \Delta t f[\hat{\mathbf{c}}(0)] \tag{13}$$

Predictor (general formula):

$$c^*(n, m, t + \Delta t) = c(n, m, t - \Delta t) + 2\Delta t f[\hat{\mathbf{c}}(t)] \tag{14}$$

Corrector:

$$c(n, m, t + \Delta t) = c(n, m, t) + \frac{\Delta t}{2}\{f[\hat{\mathbf{c}}(t)] + f[\hat{\mathbf{c}}^*(t + \Delta t)]\} \tag{15}$$

Our actual calculation of the velocities of fall of the particles relative to the liquid was done by a refinement of Stokes' law, which permits us to deal with intermediate Reynolds numbers which might arise with slurries of solids having a high density. We use an improved formula for the drag coefficient (*50*):

$$C_D = \frac{24}{R_e} + \frac{3}{R_e^{1/2}} + 0.34 \tag{16}$$

$$R_e = 2vr\rho/\eta \tag{17}$$

where v = particle velocity
r = particle radius
ρ = slurry density
η = slurry viscosity

This yields the following result for the velocity of the particle relative to the liquid:

$$v_n = \frac{2gr_n^2\Delta\rho}{9\eta}\,\frac{1}{\left[1 + \frac{1}{4}\left(\frac{\rho r_n v_n}{2\eta}\right)^{1/2} + 0.34\rho r_n v_n\right]} \tag{18}$$

where $\Delta\rho$ = particle density − slurry density

This is readily solved iteratively for v_n; one starts with the Stokes' law expression for v_n on the right-hand side.

In our analysis we have assumed perfect packing—that is, an n-particle (an agglomeration of n elementary particles) is assumed to contain no voids. This model should be valid for droplet coalescence, but certainly underestimates the volume of composite solid particles. Vold investigated a model similar to that which we are using (*51*), and found that the radius of an n-particle, r_n, is given by

$$r_n = r_1 n^{0.43} \tag{19}$$

from which we find

$$V_n = V_1 n^{1.29} \tag{20}$$

for the volume of an n-particle, and

$$\rho_n - \rho_{\text{liq}} = (\rho_l - \rho_{\text{liq}})n^{-0.29} \tag{21}$$

for the difference between the density of an n-particle and the density of the liquid phase. The slurry density is given by

$$\rho_{\text{slurry}} = \rho_{\text{liq}} + (\rho_l - \rho_{\text{liq}})V_1 \Sigma n c_n \tag{22}$$

We assume that it is liquid phase which is occluded in the composite particles.

As we shall see in the next section, solution of Eq. (12) does not lead to the rising plane of slower-settling floc remarked on by many workers, the existence of which is, according to Dixon (*47*), evidence that Class IV settling (compaction) is starting to take place. We felt that failure to observe this plane might be telling us more about the limitations of our mathematics than about physical reality, so have explored another discrete representation of Eq. (1). We wish to take account of the fact that a particle falling into a region of relatively high solids volume fraction is slowed down by the increased viscosity and decreased difference in density between the particle and the surrounding slurry. For simplicity we examine monodisperse systems; generalization to flocculating systems is straightforward, and our computer programs deal with flocculating systems.

We first approximated Eq. (1), written for monodisperse systems, by

$$\frac{\partial c}{\partial t}(m, t) = \{[v'(m-1, t) + v'(m, t)][c(m-1, t) + c(m, t)] - [v'(m, t) + v'(m+1, t)][c(m, t) + c(m+1, t)]\}/4\Delta x \tag{23}$$

Here we are simply using arithmetic averages of the particle concentrations and velocities at the centers of two adjacent slabs to calculate the flux of particles through the boundary between the slabs. This was unsuccessful; the very large concentration gradients which can occur led to large fluxes from slabs which were already nearly empty, resulting in negative concentrations. We concluded that we had best focus our attention on finding a suitable average of the velocities in our two adjacent slabs. We denote the slab velocity of a particle falling through the boundary from slab $i - 1$ into slab i by $v'_{i-1,i}$. For the case where $v'_{i-1} = v'_i$, we would expect that $v'_{i-1,i} = v'_i$. If either v'_i or v'_{i-1} is very small, we wish to have $v'_{i-1,i}$ very small. An average which satisfies both of these requirements is given by

$$v'_{i-1,i} = \frac{2v'_{i-1}v'_i}{v'_{i-1} + v'_i} \tag{24}$$

We then approximate Eq. (1), written for monodisperse systems, by

$$\frac{\partial c}{\partial t}(m, t) = [v'_{m-1,m}(t)c(m-1, t) - v'_{m,m+1}(t)c(m, t)]/\Delta x \tag{25}$$

The results obtained using this modification are discussed in the next section.

RESULTS

The model for quiescent hindered settling of flocculating particles was defined by nine input variables. These are listed in Table 1 along with the values employed in the series of runs used to determine model response. A time step and liquid density must also be included. For this study the liquid density was chosen to be 1.00 g/cm^3 and not varied.

Run 1. Run 1 was taken to be the standard run. In subsequent runs one variable at a time was changed and the results compared to the standard run. A clarifier height of 20 cm was selected in an effort to relate the simulation results to the jar test. The jar test is frequently used to assess the settling characteristics of a floc to determine a change in floc nature or prior to the design or modification of a clarification system. Figure 1 shows the settling curves generated by this model for the standard case. These were plotted as compartment number vs total mass per unit volume (in grams per cubic centimeter) every 10 sec. Table 2 describes the settling in terms of percent of the total mass found in the bottom compartment of the clarifier at various elapsed times. Over one-half the mass had settled out in 30 sec; over 90% in 110 sec. It should be noted that a compartment comprised 10% of the total clarifier volume. Table 3 presents a third means of assessing the settling characteristics. Spe-

TABLE 1

Quiescent Settling—Class 3—Input Variables

Symbol	Variable	Run numbers										
		1[a]	2	3	4	5	6	7	8	9	10[b]	11
VFRAC	Volume fraction of solids	0.001		0.005						0.130		
ETAO	Viscosity, P, g/cm-sec	0.01							0.02			
RHOS	Floc density, g/cm^3	1.05	1.10									
r	Radius of unit particle, cm	0.01										0.02
NMAX	Maximum permitted particle size	10										
AKAP	Scale factor for floc disruption rate constants, sec^{-1}	0.5				1.00						
NC	Number of compartments in clarifier	10					20					
CLARL	Height of clarifier, cm	20					40					
INDEX	Order of initial particle distribution	2			3			5				

[a] Standard run.

[b] Particle back-up run, same variables as standard run.

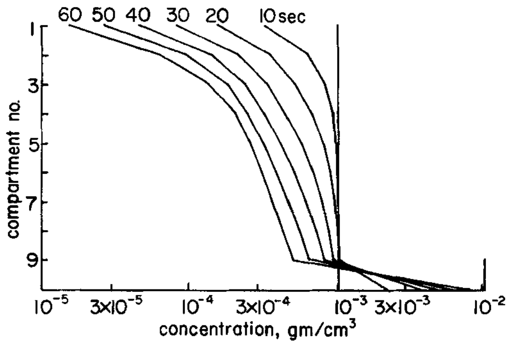

FIG. 1. Total mass per compartment vs compartment number for Run 1, standard conditions.

TABLE 2

Percent Total Mass in Last Compartment

	Run number									
Time (sec)	1	2	3	4	5	6	7	8	10	11
0	10.0	10.0	10.0	10.0	10.0	5.0	10.0	10.0	10.0	10.00
5		23.2	18.7	15.8	12.7		15.1	12.8		
10	23.2	37.2	28.0	22.4	20.9	11.6	21.2	15.5	23.2	68.0
15		50.7	37.3	29.4	26.2		28.0	18.2		
20	37.2	62.4	46.5	36.4	31.4	18.6	34.9	20.0	37.2	90.2
25		71.4	55.4	43.3	36.6		41.8			
30	50.7	77.6	63.7	49.9	41.6	25.7	48.5		50.7	95.1
35		81.9	71.1	56.1	46.4		54.8			
40	62.4	84.8	77.4	61.7	50.8	32.8	60.4		62.4	
45			82.5	66.6	54.9		65.4			
50	71.4		86.5	70.8	58.6	39.9	69.7		71.3	
55			89.5	74.2	61.8		73.3			
60	77.6		91.7	77.1	64.6	46.9	76.3		77.6	
65			93.3	79.5	67.1		78.7			
70	81.9					53.8			81.9	
80	84.8					60.2			84.8	
90	87.0					65.9			87.0	
100	88.8					70.8			88.7	
110	90.2					74.8			90.2	
120	91.6					77.9				
130	92.8									
140	93.9									
150	94.9									
160	95.7									

TABLE 3

Average Rate of Reduction of Initial Concentration, Compartments/sec

Run no.	1/2 original concentration	1/10 original concentration	1/100 original concentration
1, 6	0.153	0.051	0.038
2	0.305	0.102	N.A.
3	0.188	0.118	0.036
4	0.154	0.052	[a]
5	0.096	0.041	[a]
7	0.154	0.046	[a]

[a] Reduction not achieved by 65 sec.

cifically, this compares the average rate at which the central concentration in a compartment is reduced by 50, 90, and 99%.

Run 2. The first variable changed was the floc density. Run 2 employed a floc density of 1.10 g/cm^3 as compared to the standard 1.05 g/cm^3 value. Since the floc density appeared in the model as a difference, $\rho_{floc} - \rho_{liquid}$ (e.g., Eq. 18), this new value represented a 100% increase (viz., 1.05 − 1.00 = 0.05 vs 1.10 − 1.00 = 0.10). As anticipated, the impact was very great. Figure 2 shows the settling curves for Run 2, while Tables 2 and 3 show the effect numerically. The factor of 2 is evident in both tables. The percent settled into the bottom compartment at 20 sec in Run 1 equaled that percentage at 10 sec for Run 2; at 40 sec in Run 1, 20 sec in Run 2; etc. The reduction rate for Run 2 was twice that for Run 1 for

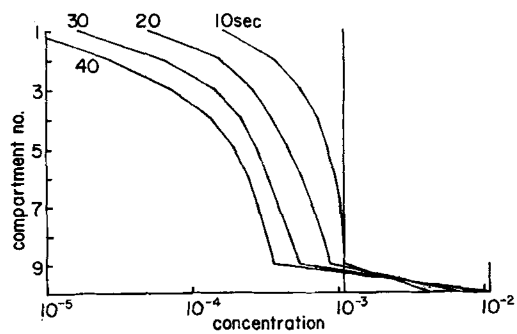

FIG. 2. Total mass per compartment vs compartment number for Run 2, $\rho_s = 1.10$.

the 1/2 and 1/10 cases. Insufficient data were generated to evaluate the reduction to 1/100 of the original compartment concentration.

Runs 3 and 9. According to Table 1, Run 3 had volume fraction solids of 0.005 compared to the standard value of 0.001. Figure 3 shows the corresponding settling curves. As anticipated, the increased solids concentration in the clarifier increased the settling rate. This reflected the increase in the rate of formation of the larger sized particles. The values listed in Table 2 show the increased percent solids in the bottom compartment in Run 3 as compared to Run 1. The corresponding increased reduction rate is seen in Table 3. However, we also anticipate that continued increase in the volume fraction will eventually have the opposite effect when a sufficiently high value is reached. The ambient viscosity is an increasing function of particle concentration (see Eq. 10). As such, the resultant increase in viscosity could offset the response from increased particle size and retard settling. We verified this in Run 9 where we employed volume fraction solids of 0.130. Table 4 summarizes these results. For a given elapsed time, even as little as 20 sec, there were appreciably fewer solids amassed in the bottom compartment for Run 9 than were amassed in either Run 1 or 3, indicating retarded settling.

Runs 4 and 7. The fourth and seventh runs varied the index used to define the particle distribution, Particles were distributed inversely proportional to their size raised to the given index (power). Increasing the distribution index weighted the population toward the smaller-sized particles. Since smaller particles have slower settling velocities, settling should be retarded. Figures 4 and 5 are the settling curves for the runs

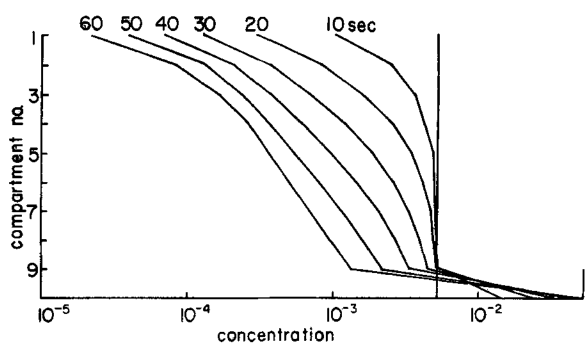

FIG. 3. Total mass per compartment vs compartment number for Run 3, VFRAC = 0.005.

TABLE 4
Summary of Volume Fraction (VFRAC) Variation Study

Parameter	Run 1	Run 3	Run 9
VFRAC	0.001	0.005	0.130
η_0	0.01	0.01	0.01
η[a]	0.010025	0.01013	0.02
Percent of Total Mass in Compartment 10			
At 0 sec	19.9	10.0	10.0
5	—	18.7	15.8
10	23.2	28.0	21.6
20	37.2	46.5	32.2[b]

[a] Calculated from Eq. (10) at time = 0 sec.
[b] Extrapolated.

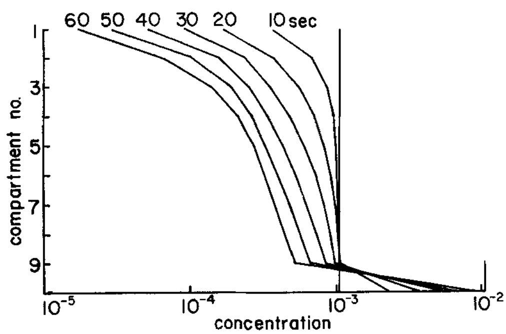

FIG. 4. Total mass per compartment vs compartment number for Run 4, index = 3.

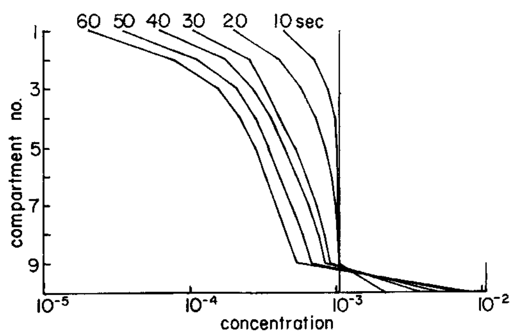

FIG. 5 Total mass per compartment vs compartment number for Run 7, index = 5.

where the index equaled 5 and 7, respectively. However, the difference was not great. This was due to the occurrence of the flocculation and disruption processes. Numerical representation of these results is found in Tables 2 and 3. The index = 3 run was only 1 or 2% lower in settled mass than the standard, index = 2, run. Similarly, the index = 3 and index = 5 runs agreed to within a few percent, the latter run correctly accumulating the slightly lower percent mass. In Table 3, Runs 1 and 3 are identical to within round-off error as is the value for the 50% reduction obtained for Run 7. A more sensitive analysis would be the evaluation and comparison of the residual mass in the first compartment for these three runs. Such residual mass can be considered "fines." Table 5 presents this evaluation. As anticipated, the higher the index, the higher the residual solids retained in the top compartment. Again, this was a result of the higher number of smaller (especially unit sized), slower falling particles initially present.

Run 5. In Run 5 the number of disruptions experienced by each particle size was increased from 0.5 to 1.0 disruptions per second (AKAP). As in Runs 4 and 7, this change weighted the population toward the smaller sizes, and slower settling was anticipated. Figure 6 shows the settling curves for Run 5. A significant retardation in settling rate did occur. This is easily verified by the numerical presentation in Tables 2 and 3. While the initial differences in particle distribution created by index variation can be reduced in time by the agglomeration/disruption pro-

TABLE 5

Comparison of Index Runs, Percent Residual Mass[a] in Compartment 1

Time (sec)	Run 1 (index = 2)	Run 4 (index = 3)	Run 7 (index = 5)
0	10.0	10.0	10.0
10	33.9	36.4	40.6
20	15.2	16.6	19.2
30	8.0	8.9	10.3
40	4.6	5.0	5.8
50	2.6	2.9	3.4
60	1.5	1.7	2.3
	% of Total Mass[b] in Compartment 10 after 60 sec		
60	77.6	77.1	76.3

[a] Percent of initial mass, i.e., in compartment at time = 0.

[b] Percent of mass from all 10 compartments.

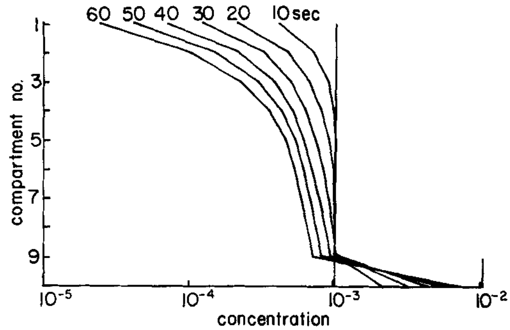

FIG. 6. Total mass per compartment vs compartment number for Run 5, $\kappa = 1.00$ disruptions/sec.

cesses, this is not the case for variations in AKAP values, since the disruption process is going on continuously during the run.

Run 6. In Run 6 two variables were simultaneously changed. These were clarifier height, CLARL, and the number of compartments, NC. This was done in order to retain the same DELTAX value (see Eq. 12, flux term), height of each compartment. Doubling the height of the clarifier essentially doubled the settling period by a factor of 2. The settling curves can be seen in Fig. 7. The increase in settling time was easily determined by comparison of the percent total solids mass in the bottom compartment at time T for Run 1 and $2T$ for Run 2 in Table 2—vis., at 20 sec, Run 1, 37.2%, and at 40 sec, Run 6, 32.8%; 40 sec, Run 1, 62.4%, and 80 sec, Run 6, 60.2%; etc. A 40-cm clarifier can also

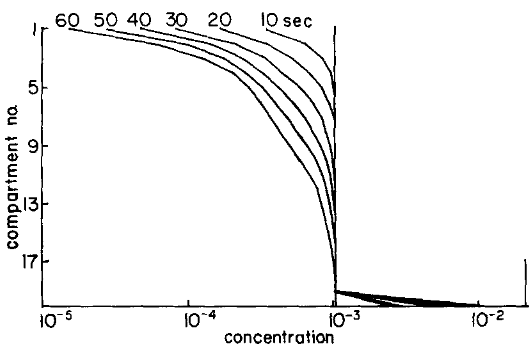

FIG. 7. Total mass per compartment vs compartment number for Run 6, clarifier length = 40 cm, number of compartments = 20.

be representative of other jar test systems. It should be noted that preliminary studies showed that increasing the number of compartments to increase the sharpness of descent linearly increased the computer time but only fractionally improved the definition of the boundary between the subsiding slurry and the supernatant. Doubling the number of compartments for a given clarifier height (i.e., halving DELTAX) only improved the sharpness of the slurry–supernatant boundary by 25%. Therefore a reasonable number of compartments (and concomitant DELTAX value) was selected without additional concern about increasing boundary sharpness. Moreover, our experience, especially with biological flocs, indicates that varying degrees of boundary definition in settling occur.

Run 8. The increase in system viscosity by increasing η_0 in Run 8 resulted in a different effect than increasing the viscosity of the system by increasing the volume fraction of solids. As anticipated, the settling time was increased by a factor of approximately 2 over the settling time determined in the standard run (η_0 = 0.01 and 0.02 P in Runs 1 and 8, respectively). Figure 8 shows a few settling curves (until time = 20 sec). The percent total mass in the last compartment for this run is slightly more than 1/2 the amount settled in the standard run. This is a ramification of the fact that the viscosity, through the definition of V_n (Eq. 18), not only affected the flux term but also the agglomeration terms (see Eq. 1, terms 1, 2, and 3).

Run 10. The same variable values were employed in Run 10 as were used in the standard run. The difference lay in the calculation of the flux, using a "reduced" velocity as discussed earlier (see Eqs. 24 and 25). We anticipated a "back-up" (i.e., increase) in solids in the penultimate (9th)

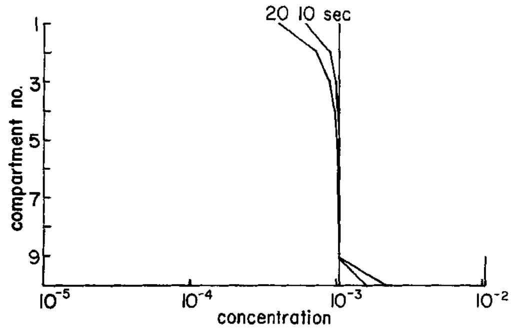

FIG. 8. Total mass per compartment vs compartment number for Run 8, $\eta_0 = 0.02$ P.

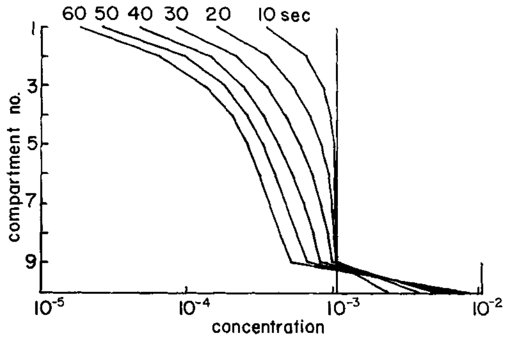

FIG. 9. Total mass per compartment vs compartment number for Run 10. Standard conditions employing "reduced velocity."

compartment as solids built up in the last, making it more difficult for additional mass to enter the last compartment. Figure 9 shows the resultant settling curves. As anticipated, the model yielded little difference in the amount of solids accumulated in the bottom compartment (see Table 2). Rather, the effect can be seen by comparing the percent solids in the 9th compartments for Runs 1 and 10. As the time elapsed increased, even at the relatively low volume fraction solids of 0.001, a difference does become evident. See Table 6. This difference increased with increasing time.

Run 11. The final value varied was the size of the unit particle radius.

TABLE 6

Percent of Total Mass in Penultimate (9th) Compartment

Time (sec)	Run 1	Run 10
0	10.0	10.0
10	10.0	10.0
20	9.9	9.9
30	9.3	9.3
40	7.9	8.0
50	6.3	6.4
60	4.9	5.0
70	3.9	4.0
80	3.3	3.4
90	2.9	3.0
100	2.6	2.7
110	2.3	2.5

TABLE 7

Particle Distribution in Compartment 10

Particle size	Number of particles per milligram of floc													
	At $t = 0$	Run 1 at 30 sec	Run 2 at 25 sec	Run 3 →	Run 5 →	Run 1 at 60 sec	Run 3 at 50 sec	Run 5 →	Run 4			Run 7		
									at 0 sec	at 25	at 50	at 0 sec	at 25	at 50
1	59.0	2.4	1.3	0.18	13.4	1.2	0.20	5.8	75.1	3.2	1.4	171.5	3.4	1.4
2	23.9	4.8	3.2	0.85	11.7	3.1	0.55	7.8	27.4	5.7	3.4	30.4	5.8	3.4
3	12.6	7.0	5.1	1.9	12.2	4.9	1.3	9.6	12.8	7.8	5.3	11.2	8.0	5.4
4	7.2	6.6	5.9	3.9	6.8	6.1	3.1	7.0	6.5	6.7	6.2	4.3	6.7	6.3
5	4.3	7.2	6.9	6.5	6.5	7.2	6.2	7.1	3.2	7.1	7.2	1.6	7.1	7.2
6	2.5	2.3	2.1	1.7	2.4	2.2	1.6	2.4	1.5	2.4	2.2	0.54	2.4	2.2
7	1.3	2.1	2.0	1.9	1.8	2.1	1.8	2.0	0.68	2.1	2.1	0.15	2.1	2.1
8	0.67	2.0	2.0	2.4	1.4	2.1	2.4	1.7	0.26	1.9	2.1	3.1×10^{-2}	1.9	2.1
9	0.27	3.8	4.2	6.9	2.0	4.6	7.8	2.8	6.7×10^{-2}	3.4	4.4	3.6×10^{-3}	3.4	4.4
10	5.9×10^{-2}	5.4	5.5	6.6	3.8	5.8	6.7	4.6	7.5×10^{-3}	5.2	5.7	1.0×10^{-4}	5.2	5.7

Increasing the unit particle size had a dramatic effect, greatly reducing the settling time. As seen in Table 2, after only 30 sec over 95% of the total system's mass had entered the bottom compartment. This compared with only 50.7% in the standard run. When only 10 sec had elapsed, 68% had reached bottom. In the same period only 23.2% had fallen in the Run 1 simulation.

Because of the continuous flocculation/disruption processes accompanying the particle flux, the particle size distribution in each compartment was continuously changing. We present in Table 7 particle distributions (in Compartment 10) for Runs 1–5 and 7 at time 0, 25, and 50 sec. In order to facilitate meaningful comparisons, particle populations are reported per milligram of floc.

As described earlier, varying the index will change the initial size distribution. Runs 4 and 7 vary the index value. The increase in the smaller-sized particle population is evident when indices 2, 3, and 5 (Runs 1, 4, and 7, respectively) are compared. As mentioned earlier, the impact of the flocculation/disruption processes overwhelmed these initial differences. At 50 sec the population distributions of Runs 4 and 7 were essentially identical and very similar to the Run 1 distribution (which was evaluated at 60 sec). The increase in volume fraction (Run 3) resulted in slightly higher numbers of larger particles in comparison to Run 1. This is seen at both comparison times. As stated before, it is this increase which shortened the settling time. The increase in smaller (sizes 1–4) particles in Run 5 correctly reflected the increased disruption rate over Run 1 at both time comparisons. The comcomitant decrease of the concentrations of the larger particles (8–10) was also successfully modeled.

This model for quiescent hindered settling of flocculating slurries predicts the expected response of the system to variation in input parameters. These parameters include particle density, viscosity, particle size, clarifier height, percent solids, particle size distribution, and particle disruption rates. The computer program analyzing the model can be run on a relatively small computer in 10 min or less. It should be noted that the model can also be used for larger-sized batch clarifiers (on the order of 1 m and taller). The model only requires that the proper variable values be selected to describe the system under consideration.

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